

First Grade

2018-2019 Guide

February 11th - April 5th

Eureka

Module 4: *Place Value, Comparison, Addition and Subtraction to 40*



ORANGE PUBLIC SCHOOLS

OFFICE OF CURRICULUM AND INSTRUCTION

OFFICE OF MATHEMATICS

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Module 4 Performance Overview

- In Topic A, students study, organize, and manipulate numbers within 40. Having worked with creating a ten and some ones in Module 2, students now recognize multiple tens and ones. Students use fingers, linking cubes, dimes, and pennies to represent numbers to 40 in various ways—from all ones to tens and ones. They use a place value chart to organize units. The topic closes with the identification of 1 more, 1 less, 10 more, and 10 less as students learn to add or subtract like units
- In Topic B, students compare quantities and begin using the symbols for greater than ($>$) and less than ($<$). Students demonstrate their understanding of place value when they recognize that 18 is less than 21 since 2 tens already have a greater value than 1 ten 8 ones. To support understanding, the first lesson in the topic focuses on identifying the greater or less than amount. With this understanding, students label each of the quantities being compared and compare from left to right.
- Topic C focuses on addition and subtraction of tens. With this understanding, students add and subtract a multiple of 10 from another multiple of 10. The topic closes with the addition of multiples of 10 to numbers less than 40 (e.g., $12 + 30$).
- In Topic D, students use familiar strategies to add two-digit and single-digit numbers within 40. Students apply the Level 2 strategy of counting on and use the Level 3 strategy of making ten, this time making the next ten. For instance, when adding $28 + 5$, students break 5 into 2 and 3 so that 28 and 2 can make the next ten, which is 30, or 3 tens, and then add 3 to make 33. The topic closes with students sharing and critiquing peer strategies.
- In Topic E, students consider new ways to represent larger quantities when approaching put together/take apart with total or addend unknown and add to with result or change unknown word problems. Students begin labeling drawings with numerals and eventually move to tape diagrams to represent the problems pictorially. Throughout this topic, students continue developing their skills with adding single-digit and double-digit numbers.
- The module closes with Topic F, focusing on adding like place value units as students add two-digit numbers. The topic begins with interpreting two-digit numbers in varied combinations of tens and ones (e.g., $34 = 34$ ones = 3 tens 4 ones = 2 tens 14 ones = 1 ten 24 ones). This flexibility in representing a given number prepares students for addition with regrouping (e.g., $12 + 8 = 1$ ten 10 ones = 2 tens or $18 + 16 = 2$ tens 14 ones = 3 tens 4 ones). To close the module, students add pairs of numbers with varied sums in the ones place to support flexibility in thinking.



**Module 4: Place Value, Comparison,
Addition and Subtraction to 40**

Pacing:

February 11th – April 5th

35 Days

Topic	Lesson	Student Lesson Objective/ Supportive Videos
Topic A: Tens and Ones	Lesson 1	Compare the efficiency of counting by ones and counting by tens https://www.youtube.com/watch?v
	Lesson 2	Use the place value chart to record and name tens and ones within a two-digit number https://www.youtube.com/watch?v
	Lesson 3	Interpret two-digit numbers as either tens and some ones or as all ones. https://www.youtube.com/watch?v
	Lesson 4	Write and interpret two-digit numbers as addition sentences that combine tens and ones https://www.youtube.com/watch?v
	Lesson 5	Identify 10 more, 10 less, 1 more, and 1 less than a two-digit number https://www.youtube.com/watch?v
	Lesson 6	Use dimes and pennies as representations of tens and ones https://www.youtube.com/watch?v
Topic B: Comparison of Two-Digit Numbers	Lesson 7	Compare two quantities, and identify the greater or lesser of the two given numerals. https://www.youtube.com/watch?v
	Lesson 8	Compare quantities and numerals from left to right https://www.youtube.com/watch?v
	Lesson 9 & 10	Use the symbols $>$, $=$, and $<$ to compare quantities and numerals https://www.youtube.com/watch?v https://www.youtube.com/watch?v
Topic C: Addition and Subtraction of Tens	Lesson 11	Add and subtract tens from a multiple of 10. https://www.youtube.com/watch?v
	Lesson 12	Add tens to a two-digit number. https://www.youtube.com/watch?v

**Mid-Module Assessment Task
(Interview Style)**

February 26-28th

<p>Topic D: Addition of Tens or Ones to a Two-Digit Number</p>	Lesson 13 &14	Use counting on and the make ten strategy when adding across a ten. https://www.youtube.com/watch?v https://www.youtube.com/watch?v
	Lesson 15	Use single-digit sums to support solutions for analogous sums to 40 https://www.youtube.com/watch?v
	Lesson 16 &17	Add ones and ones or tens and tens https://www.youtube.com/watch?v https://www.youtube.com/watch?v
	Lesson 18	Share and critique peer strategies for adding two-digit numbers. https://www.youtube.com/watch?v
<p>Topic E: Varied Problem Types within 20</p>	Lesson 19	Use tape diagrams as representations to solve put together/take apart with total unknown and add to with result unknown word problems. https://www.youtube.com/watch?v
	Lesson 20 &21	Recognize and make use of part-whole relationships within tape diagrams when solving a variety of problem types https://www.youtube.com/watch?v https://www.youtube.com/watch?v
	Lesson 22	Write word problems of varied types https://www.youtube.com/watch?v
	Lesson 22	Write word problems of varied types https://www.youtube.com/watch?v
<p>Topic F: Addition of Tens and Ones to a Two-Digit</p>	Lesson 23	Interpret two-digit numbers as tens and ones, including cases with more than 9 ones https://www.youtube.com/watch?v
	Lesson 24 &25	Add a pair of two-digit numbers when the ones digits have a sum less than or equal to 10 https://www.youtube.com/watch?v

Number		https://www.youtube.com/watch?v
	Lesson 26 &27	Add a pair of two-digit numbers when the ones digits have a sum greater than 10 https://www.youtube.com/watch?v https://www.youtube.com/watch?v
	Lesson 26 &27	Add a pair of two-digit numbers when the ones digits have a sum greater than 10 https://www.youtube.com/watch?v
Lesson 28-29	Add a pair of two-digit numbers with varied sums in the ones https://www.youtube.com/watch?v	

End-of- Module Assessment Task

(Interview Style: 3 days)

March 27-29th 2019

NJSL Standards:

Module 4: Place Value, Comparison, Addition and Subtraction to 40

1.OA.1

Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

Introduce situations for students to model- starting with concrete materials; and as students are ready- working with drawings, part-part- whole representations, and number lines.

Have a variety of materials for students to model the problems.

First grade students extend their experiences in Kindergarten by working with numbers to 20 to solve a new type of problem situation: Compare (See **Table 1** in this document for examples of all problem types). In a Compare situation, two amounts are compared to find “How many more” or “How many less”.

As students show proficiency with models and drawings, begin to demonstrate how to represent the actions using equations. Give students the opportunity to say the equation orally and then in writing.

The vocabulary of comparison situations can cause confusion for students. While the words *more than* implies addition and *fewer than* implies subtraction, that is not always the case. Avoid teaching “ Key Words”

Problem Type: Compare		
<p><u>Difference Unknown:</u> “How many more?” version. Lucy has 7 apples. Julie as 9 apples. How many more apples does Julie have than Lucy?</p>	<p><u>Bigger Unknown:</u> “More” version suggests operation. Julie has 2 more apples than Lucy. Lucy has 7 apples. How many apples does Julie have?</p>	<p><u>Smaller Unknown:</u> Version with “more” Mastery expected in Second Grade</p>
<p><u>“How many fewer?” version</u> Lucy as 7 apples. Julie has 9 apples. How many fewer apples does Lucy have than Julie? $7 + \square = 9$ $9 - 7 = \square$</p>	<p><u>Bigger Unknown:</u> Version with “fewer” Mastery expected in Second Grade</p>	<p><u>Smaller Unknown:</u> “Fewer” version suggests operation. Lucy has 2 fewer apples than Julie. Julie has 9 apples. How many apples does Lucy have?</p>

1.NBT.1

Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral.

First Grade students rote count forward to 120 by counting on from any number less than 120. First graders develop accurate counting strategies that build on the understanding of how the numbers in the counting sequence are related—each number is one more (or one less) than the number before (or after). In addition, first grade students read and write numerals to represent a given amount.

As first graders learn to understand that the position of each digit in a number impacts the quantity of the number, they become more aware of the order of the digits when they write numbers. For example, a student may write “17” and mean “71”. Through teacher demonstration, opportunities to “find mistakes”, and questioning by the teacher (“I am reading this and it says seventeen. Did you mean seventeen or seventy-one? How can you change the number so that it reads seventy-one?”), students become precise as they write numbers to 120.

Students should have ample experiences with the hundreds chart to see patterns between numbers, such as all of the numbers in a column on the hundreds chart have the same digit in the ones place, and all of the numbers in a row have the same digit in the tens place

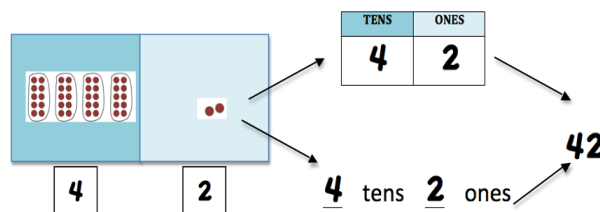
1.NBT.2

Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases:

a. 10 can be thought of as a bundle of ten ones- called a “ten”

c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones)

First Grade students are introduced to the idea that a bundle of ten ones is called a “ten”. This is known as “unitizing”. When first grade students unitize a group of ten as a whole unit (“a ten”), they are able to count groups as though they were individual objects. This is a monumental shift in thinking and can often be challenging young children to consider a group of something as “one” when all previous experiences have been counting single objects. This is the foundation of the place value system and requires time and rich experiences with concrete manipulatives to develop.



Make sure to reinforce the concept that 4 tens is the same as 40. Students should be asked to represent both ways. The use of hide zero cards will help solidify this thinking.

A student's ability to conserve number is an important aspect of this standard. Therefore, first graders require ample time grouping proportional objects (e.g., cubes, beans, bead, ten-frames) to make groups of ten, rather than using only pre-grouped materials (e.g., Base 10 Blocks, pre-made ben sticks) that have to be "traded" or are non-proportional (e.g., money, place value disks)

Students should explore the idea that decade numbers (e.g 10, 20, 30, 40,etc) are groups of ten with no left over ones.

It is best to make a ten with unifix cubes or other materials that students can group.

As students are representing the various amounts, it is important that an emphasis is placed on the language associated with the quantity.

1.NBT.3

Compare two two-digit numbers based on meanings of the ten and ones digits, recording the results of comparisons with the symbols $<$, $>$, $=$

First Graders use their understanding of groups and order digits to compare two numbers by examining the amount of tens and ones in each number

Students are introduced to the symbols greater than ($>$), less than ($<$) and equal to ($=$)

Language such as "The alligator eats the bigger number" is not mathematical and should be avoided

Students should have ample experiences communicating their comparisons using words, models and in context before using only symbols in this standard.

Example: 42 _ 45

Student: 42 has 4 tens and 2 ones. 45 has 4 tens and 5 ones. They have the same number of tens but 45 has more ones than 42. So, 45 is greater than 42. So $42 < 45$.

1.NBT.4

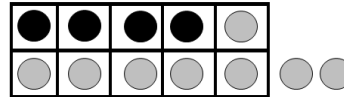
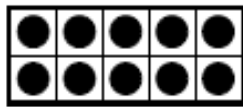
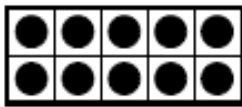
Add within 100, including adding a two-digit number and a one digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.

First Grade students use concrete materials, models, drawings and place value strategies to add within 100. They do so by being flexible with numbers as they use the base-ten system to solve problems. The standard algorithm of carrying or borrowing is neither an expectation nor a focus in First Grade. Students use strategies for addition and subtraction in Grades K-3. By the end of Third Grade students use a range of algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction to fluently add and subtract within 1000.

Students are expected to fluently add and subtract multi-digit whole numbers using the standard algorithm by the end of Grade 4

Example: 24 red apples and 8 green apples are on the table. How many apples are on the table?

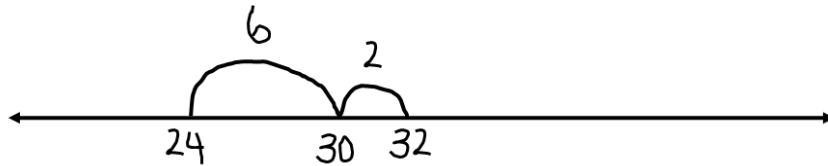
Student A: I used ten frames. I put 24 chips on 3 ten frames. Then, I counted out 8 more chips. 6 of them filled up the third ten frame. That meant I had 2 left over. 3 tens and 2 left over. That's 32. So, there are 32 apples on the table.



Student B:

I used an open number line. I started at 24. I knew that I needed 6 more jumps to get to 30. So, I broke apart 8 into 6 and 2. I took 6 jumps to land on 30 and then 2 more. I landed on 32. So, there are 32 apples on the table.

$24 + 6 =$ 30 $30 + 2 =$



Student C:

I turned 8 into 10 by adding 2 because it's easier to add. So, 24 and ten more is 34. But, since I added 2 extra, I had to take them off again. 34 minus 2 is 32. There are 32 apples on the table.

Include problems that provide a context for addition as often as possible (Table 1)

Equations should be written both horizontally and vertically.

Encourage students to make estimates before adding to determine if their answers are reasonable.

Pose questions that require students to think about the strategies they are using to add, making connections to place value

1.NBT.5

Given a two-digit number, mentally find 10 more or 10 less than the number without having to count; explain the reasoning used

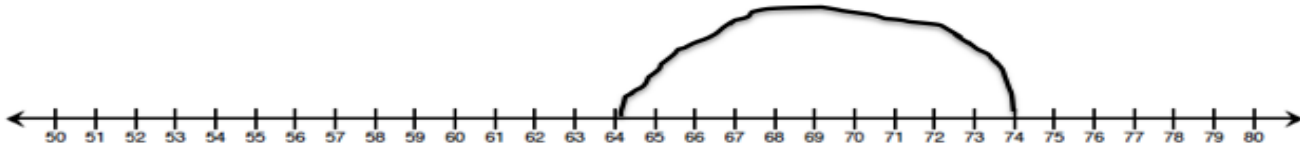
First graders build on their counting by tens work in Kindergarten by mentally adding ten more and ten less than with any number less than 100

First graders are not expected to compute differences of two-digit numbers other than multiples of ten

Ten frames, hundred charts, and the number line are powerful tools that students can use to model adding and subtracting tens with the goal of internalizing the relationships and solving these types of problems mentally

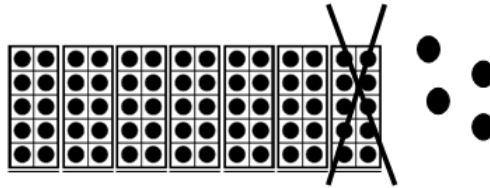
Example: There are 74 birds in the park. 10 birds fly away. How many birds are in the park now?

Student A: I thought about a number line. I started at 74. Then, because 10 birds flew away, I took a leap of 10. I landed on 64. So, there are 64 birds left in the park



Student B

I pictured 7 ten frames and 4 left over in my head. Since 10 birds flew away, I took one of the ten frames away. That left 6 ten frames and 4 left over. So, there are 64 birds left in the park



1.NBT.6

Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

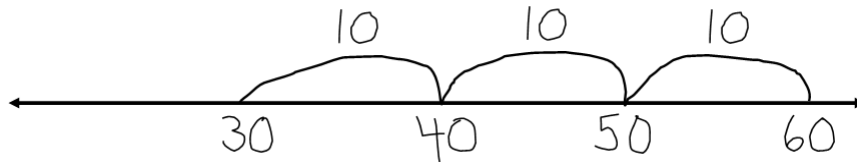
First Graders use concrete models, drawings and place value strategies to subtract multiples of 10 from decade numbers (e.g. 30, 40, 50). These opportunities develop fluency of additions and subtraction and reinforce counting up and back by 10s.

Ask students to look for patterns and explain their work

Example: There are 60 students in the gym. 30 students leave. How many students are still in the gym?

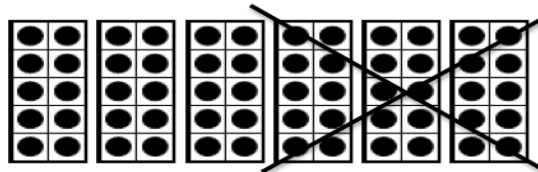
Student A

I used a number line. I started at 60 and moved back 3 jumps of 10 and landed on 30. There are 30 students left.



Student B

I used ten frames. I had 6 ten frames- that's 60. I removed three ten frames because 30 students left the gym. There are 30 students left in the gym.



M : Major Content

S: Supporting Content

A : Additional Content

Common addition and subtraction.¹

	RESULT UNKNOWN	CHANGE UNKNOWN	START UNKNOWN
ADD TO	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? $2 + 3 = ?$	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? $2 + ? = 5$	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? $? + 3 = 5$
TAKE FROM	Five apples were on the table. I ate two apples. How many apples are on the table now? $5 - 2 = ?$	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? $5 - ? = 3$	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? $? - 2 = 3$
	TOTAL UNKNOWN	ADDEND UNKNOWN	BOTH ADDENDS UNKNOWN²
PUT TOGETHER / TAKE APART³	Three red apples and two green apples are on the table. How many apples are on the table? $3 + 2 = ?$	Five apples are on the table. Three are red and the rest are green. How many apples are green? $3 + ? = 5$, $5 - 3 = ?$	Grandma has five flowers. How many can she put in the red vase and how many in her blue vase? $5 = 0 + 5$, $5 + 0 = 5$, $5 = 1 + 4$, $5 = 4 + 1$, $5 = 2 + 3$, $5 = 3 + 2$
COMPARE	DIFFERENCE UNKNOWN	BIGGER UNKNOWN	SMALLER UNKNOWN
	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? $2 + ? = 5$, $5 - 2 = ?$	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? $2 + 3 = ?$, $3 + 2 = ?$	(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? $5 - 3 = ?$, $? + 3 = 5$

¹ Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

² These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean, makes or results in but always does mean is the same number as.

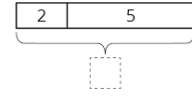
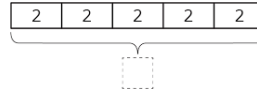
³ Either addend can be unknown, so there are three variations of these problem situations. Both addends Unknown is a productive extension of the basic situation, especially for small numbers less than or equal to 10.

⁴ For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

<http://www.corestandards.org/Math/Content/mathematics-glossary/Table-1/>

Teaching Representations/ Manipulatives/ Tools:

- Arrow notation
- Comparison symbols: >, <, =
- Dime
- Hide Zero cards
- Hundred chart
- Number bond
- Tape Diagram
- Penny
- Place value chart
- Quick Ten
- Rekenrek



$$\begin{array}{l}
 34 \xrightarrow{+1} 35 \qquad 34 \xrightarrow{-1} 33 \\
 34 \xrightarrow{+10} 44 \qquad 34 \xrightarrow{-10} 24 \\
 \text{arrow notation}
 \end{array}$$

120 Chart										120 Chart									
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120

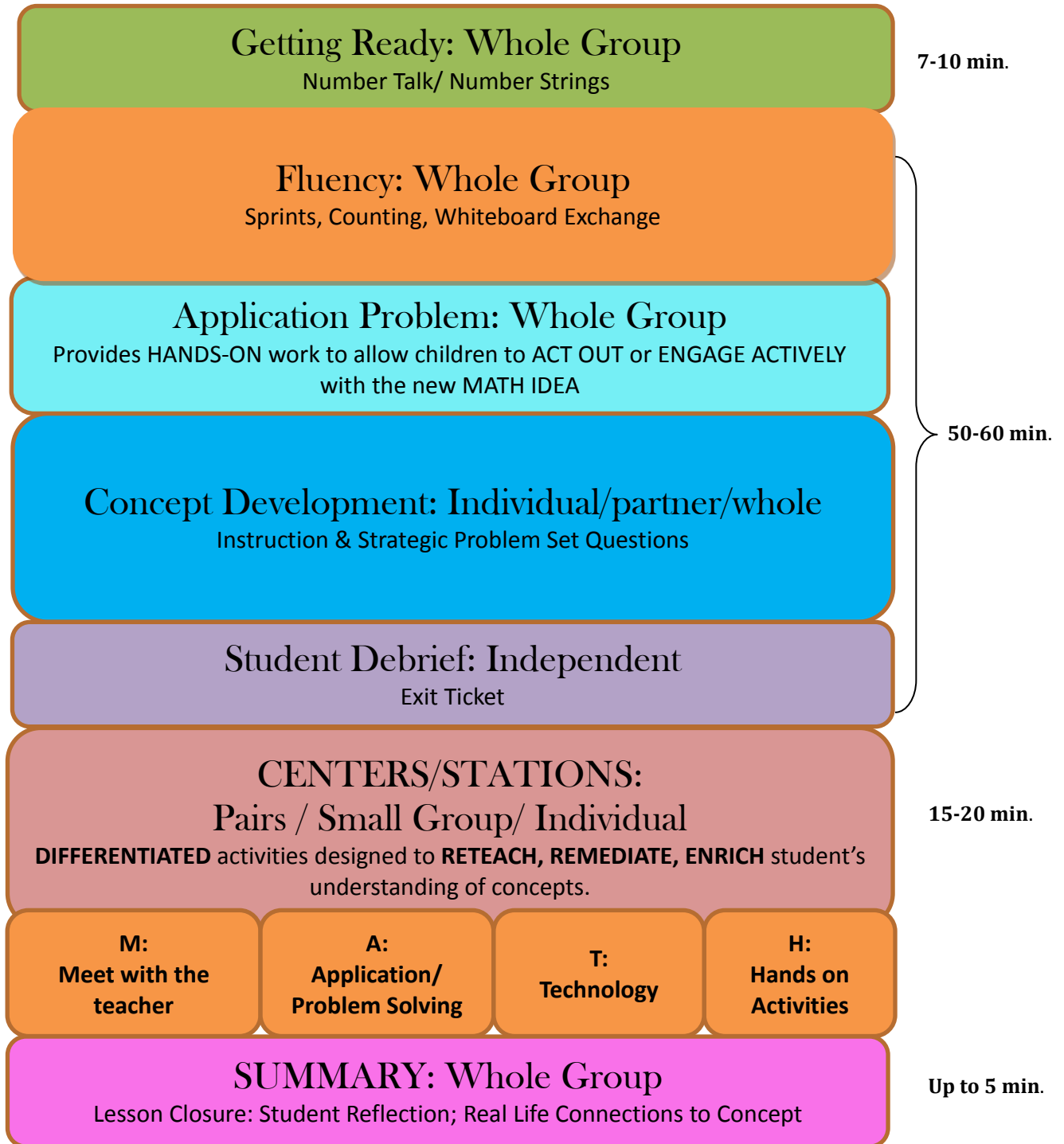
Terminology/ Symbols

- > (greater than)
- < (less than)
- Place value (quantity represented by a digit in a particular place within a number)

**Module 3 Assessment / Authentic Assessment
Recommended Framework**

Assessment	CCSS	Estimated Time	Format
<p><u>Eureka Math</u> Module 4: Place Value, Comparison, Addition and Subtraction to 40</p>			
Portfolio/Authentic Assessment: Pet Snake	1.OA.1	30 mins	Individual
Portfolio/Authentic Assessment: 20 Tickets	1.OA.1	30 mins	Individual
Optional Mid-Module Assessment	1.NBT.1 1.NBT.2 1.NBT.3 1.NBT.4 1.NBT.5 1.NBT.6	1 Block	Individual
Optional End of Module Assessment	1.OA.1 1.NBT.1 1.NBT.2 1.NBT.3 1.NBT.4 1.NBT.6 1.NBT.6	1 Block	Individual

First Grade Ideal Math Block



Eureka Lesson Structure:

Fluency:

- Sprints
- Counting : Can start at numbers other than 0 or 1 and might include supportive concrete material or visual models
- Whiteboard Exchange

Application Problem:

- Engage students in using the RDW Process
- Sequence problems from simple to complex and adjust based on students' responses
- Facilitate share and critique of various explanations, representations, and/or examples.

Concept Development: (largest chunk of time)

Instruction:

- Maintain overall alignment with the objectives and suggested pacing and structure.
- Use of tools, precise mathematical language, and/or models
- Balance teacher talk with opportunities for peer share and/or collaboration
- Generate next steps by watching and listening for understanding

Problem Set: (Individual, partner, or group)

- Allow for independent practice and productive struggle
- Assign problems strategically to differentiate practice as needed
- Create and assign remedial sequences as needed

Student Debrief:

- Elicit students thinking, prompt reflection, and promote metacognition through student centered discussion
- Culminate with students' verbal articulation of their learning for the day
- Close with completion of the daily Exit Ticket (opportunity for informal assessment that guides effective preparation of subsequent lessons) as needed.

PARCC Assessment Evidence/Clarification Statements

CCSS	Evidence Statement	Clarification	Math Practices
1.OA.A.1	Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart and comparing, with unknown in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.	i) Tasks should include all problem situations and all of their subtypes and language variants. Mastery is expected in “Add To” and “Take From” - Result and Change Unknown Problems, “Put Together/Take Apart” Problems, “Compare” – Difference Unknown, Bigger Unknown (more version) and Smaller Unknown (fewer version) Problems (for more information see CCSS Table 1 and OA Progression, p. 9.) ii) Interviews (individual or small group) are used to assess mastery of different problem types.	MP 1, 4
1.NBT.2-1	Understand that the two digits of a two-digit number represent amounts of tens and ones.	i)Tasks should focus on the understanding of two-digit numbers as some number of “tens” and some number of “ones.” ii) Interviews (individual or small group) should target this understanding	MP 7,8
1.NBT.2-2	Understand that 10 can be thought of as a bundle of ten ones — called a “ten.”.	i)Tasks should focus on the understanding of ten “ones” as a unit of one “ten.” ii) Interviews (individual or small group) should target this understanding.	MP 7,8

Number Talks Cheat Sheet

What does Number Talks look like?

- Students are near each other so they can communicate with each other (central meeting place)
- Students are mentally solving problems
- Students are given thinking time
- Thumbs up show when they are ready
- Teacher is recording students' thinking

Communication

- Having to talk out loud about a problem helps students clarify their own thinking
- Allow students to listen to other's strategies and value other's thinking
- Gives the teacher the opportunity to hear student's thinking

Mental Math

- When you are solving a problem mentally you must rely on what you know and understand about the numbers instead of memorized procedures
- You must be efficient when computing mentally because you can hold a lot of quantities in your head

Thumbs Up

- This is just a signal to let you know that you have given your students enough time to think about the problem
- It will give you a picture of who is able to compute mentally and who is struggling
- It isn't as distracting as a waving hand

Teacher as Recorder

- Allows you to record students' thinking in the correct notation
- Provides a visual to look at and refer back to
- Allows you to keep a record of the problems posed and which students offered specific strategies

Purposeful Problems

- Start with small numbers so the students can learn to focus on the strategies instead of getting lost in the numbers
- Use a number string (a string of problems that are related to and scaffold each other)

Starting Number Talks in your Classroom

- Start with specific problems in mind
- Be prepared to offer a strategy from a previous student
- It is ok to put a student's strategy on the backburner
- Limit your number talks to about 15 minutes
- Ask a question, don't tell!

The teacher asks questions:

- Who would like to share their thinking?
- Who did it another way?
- How many people solved it the same way as Billy?
- Does anyone have any questions for Billy?
- Billy, can you tell us where you got that 5?
- How did you figure that out?
- What was the first thing your eyes saw, or your brain did?
- What are Number Talks and Why are they

Student Name: _____

Task: _____

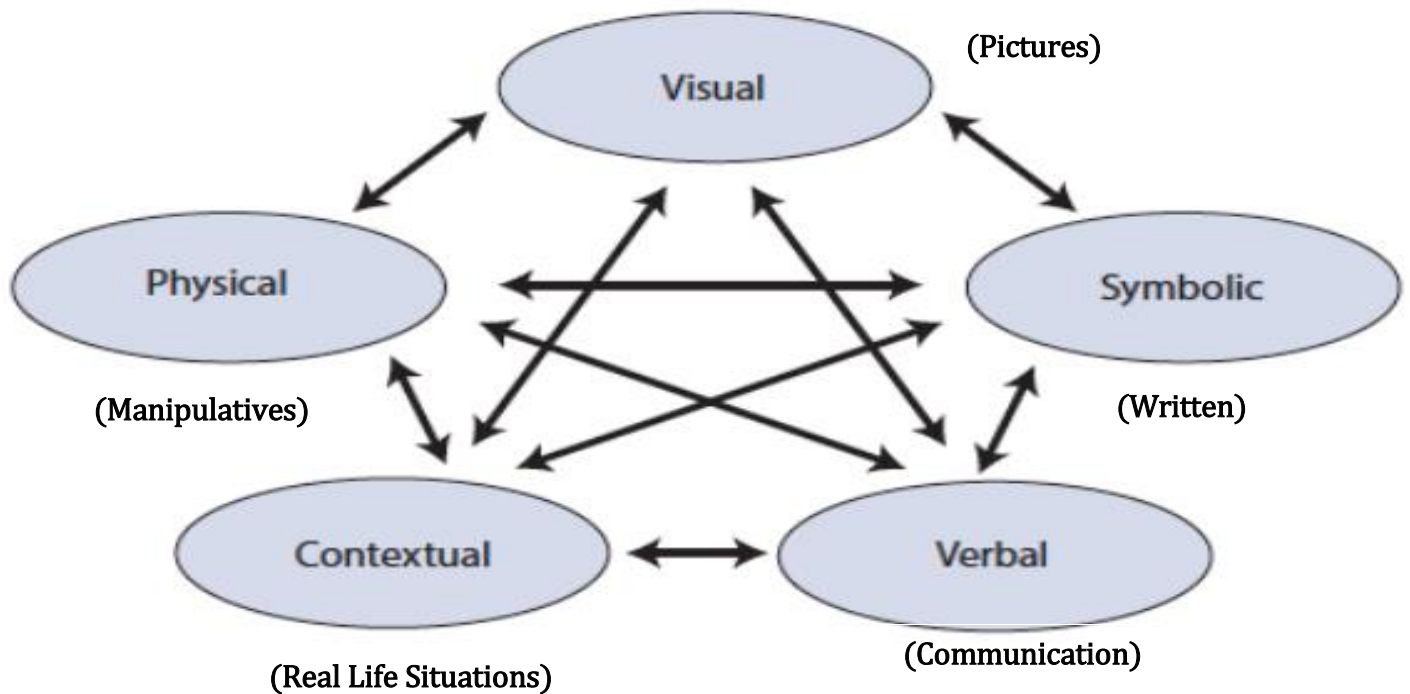
School: _____

Teacher: _____

Date: _____

"I CAN....."	STUDENT FRIENDLY RUBRIC				SCORE
	...a start 1	...getting there 2	...that's it 3	WOW! 4	
Understand	I need help.	I need some help.	I do not need help.	I can help a classmate.	
Solve	I am unable to use a strategy.	I can start to use a strategy.	I can solve it more than one way.	I can use more than one strategy and talk about how they get to the same answer.	
Say or Write	I am unable to say or write.	I can write or say some of what I did.	I can write and talk about what I did. I can write or talk about why I did it.	I can write and say what I did and why I did it.	
Draw or Show	I am not able to draw or show my thinking.	I can draw, but not show my thinking; or I can show but not draw my thinking;	I can draw and show my thinking	I can draw, show and talk about my thinking.	

Use and Connection of Mathematical Representations



The Lesh Translation Model

Each oval in the model corresponds to one way to represent a mathematical idea.

Visual: When children draw pictures, the teacher can learn more about what they understand about a particular mathematical idea and can use the different pictures that children create to provoke a discussion about mathematical ideas. Constructing their own pictures can be a powerful learning experience for children because they must consider several aspects of mathematical ideas that are often assumed when pictures are pre-drawn for students.

Physical: The manipulatives representation refers to the unifix cubes, base-ten blocks, fraction circles, and the like, that a child might use to solve a problem. Because children can physically manipulate these objects, when used appropriately, they provide opportunities to compare relative sizes of objects, to identify patterns, as well as to put together representations of numbers in multiple ways.

Verbal: Traditionally, teachers often used the spoken language of mathematics but rarely gave students opportunities to grapple with it. Yet, when students do have opportunities to express their mathematical reasoning aloud, they may be able to make explicit some knowledge that was previously implicit for them.

Symbolic: Written symbols refer to both the mathematical symbols and the written words that are associated with them. For students, written symbols tend to be more abstract than the other representations. I tend to introduce symbols after students have had opportunities to make connections among the other representations, so that the students have multiple ways to connect the symbols to mathematical ideas, thus increasing the likelihood that the symbols will be comprehensible to students.

Contextual: A relevant situation can be any context that involves appropriate mathematical ideas and holds interest for children; it is often, but not necessarily, connected to a real-life situation.

The Lesh Translation Model: Importance of Connections

As important as the ovals are in this model, another feature of the model is even more important than the representations themselves: The arrows! The arrows are important because they represent the connections students make between the representations. When students make these connections, they may be better able to access information about a mathematical idea, because they have multiple ways to represent it and, thus, many points of access.

Individuals enhance or modify their knowledge by building on what they already know, so the greater the number of representations with which students have opportunities to engage, the more likely the teacher is to tap into a student's prior knowledge. This "tapping in" can then be used to connect students' experiences to those representations that are more abstract in nature (such as written symbols). Not all students have the same set of prior experiences and knowledge. Teachers can introduce multiple representations in a meaningful way so that students' opportunities to grapple with mathematical ideas are greater than if their teachers used only one or two representations.

Concrete Pictorial Abstract (CPA) Instructional Approach

The CPA approach suggests that there are three steps necessary for pupils to develop understanding of a mathematical concept.

Concrete: “Doing Stage”: Physical manipulation of objects to solve math problems.

Pictorial: “Seeing Stage”: Use of imaged to represent objects when solving math problems.

Abstract: “Symbolic Stage”: Use of only numbers and symbols to solve math problems.

CPA is a gradual systematic approach. Each stage builds on to the previous stage. Reinforcement of concepts are achieved by going back and forth between these representations and making connections between stages. Students will benefit from seeing parallel samples of each stage and how they transition from one to another.

Read, Draw, Write Process

READ the problem. Read it over and over.... And then read it again.

DRAW a picture that represents the information given. During this step students ask themselves: Can I draw something from this information? What can I draw? What is the best model to show the information? What conclusions can I make from the drawing?

WRITE your conclusions based on the drawings. This can be in the form of a number sentence, an equation, or a statement.

Students are able to draw a model of what they are reading to help them understand the problem. Drawing a model helps students see which operation or operations are needed, what patterns might arise, and which models work and do not work. Students must dive deeper into the problem by drawing models and determining which models are appropriate for the situation.

While students are employing the RDW process they are using several Standards for Mathematical Practice and in some cases, all of them.

Mathematical Discourse and Strategic Questioning

Discourse involves asking strategic questions that elicit from students their understanding of the context and actions taking place in a problem, how a problem is solved and why a particular method was chosen. Students learn to critique their own and others' ideas and seek out efficient mathematical solutions.

While classroom discussions are nothing new, the theory behind classroom discourse stems from constructivist views of learning where knowledge is created internally through interaction with the environment. It also fits in with socio-cultural views on learning where students working together are able to reach new understandings that could not be achieved if they were working alone.

Underlying the use of discourse in the mathematics classroom is the idea that mathematics is primarily about reasoning not memorization. Mathematics is not about remembering and applying a set of procedures but about developing understanding and explaining the processes used to arrive at solutions.

Teacher Questioning:

Asking better questions can open new doors for students, promoting mathematical thinking and classroom discourse. Can the questions you're asking in the mathematics classroom be answered with a simple “yes” or “no,” or do they invite students to deepen their understanding?

The most
important thing
is to NEVER
stop
questioning

Albert Einstein

To help you encourage deeper discussions, here are 100 questions to incorporate into your instruction by Dr. Gladis Kersaint, mathematics expert and advisor for Ready Mathematics.

100 questions that promote

Mathematical Discourse

Help students **work together** to make sense of mathematics

- 1 What **strategy** did you use?
- 2 Do you **agree**?
- 3 Do you **disagree**?
- 4 Would you **ask the rest of the class** that question?
- 5 Could you **share your method** with the class?
- 6 What part of what he said **do you understand**?
- 7 Would someone like to **share** ___?
- 8 Can you **convince the rest of us** that your answer makes sense?
- 9 **What do others think** about what [student] said?
- 10 Can someone **retell or restate** [student]'s explanation?
- 11 Did you **work together**? In what way?
- 12 Would anyone like to **add to what was said**?
- 13 Have you **discussed** this with your group? With others?
- 14 Did anyone get a **different answer**?
- 15 **Where** would you go for **help**?
- 16 **Did everybody get a fair chance** to talk, use the manipulatives, or be the recorder?
- 17 How could you help another student **without telling them the answer**?
- 18 **How would you explain** ___ to someone who missed class today?

Help students **rely more on themselves** to determine whether something is mathematically correct

- 19 Is this a **reasonable answer**?
- 20 Does that make **sense**?
- 21 **Why** do you think that? Why is that true?
- 22 Can you **draw a picture or make a model** to show that?
- 23 **How** did you reach that conclusion?
- 24 Does anyone want to **revise** his or her answer?
- 25 **How were you sure** your answer was right?

Ready

Help students learn to reason mathematically

- 26 How did you **begin** to think about this problem?
- 27 What is **another way** you could solve this problem?
- 28 How could you **prove** _____?
- 29 Can you **explain how your answer is different from or the same as** [student]'s answer?
- 30 Let's **break the problem into parts**. What would the parts be?
- 31 Can you **explain this part more specifically**?
- 32 Does that **always work**?
- 33 Can you think of a case where that **wouldn't work**?
- 34 How did you **organize** your information? Your thinking?

Help students with problem comprehension

- 39 What is this problem about? What can you **tell me about it**?
- 40 Do you need to **define or set limits** for the problem?
- 41 How would you **interpret** that?
- 42 Could you **reword that in simpler terms**?
- 43 Is there something that can be **eliminated** or that is **missing**?
- 44 Could you **explain** what the problem is asking?
- 45 What **assumptions** do you have to make?
- 46 What do you **know** about this part?
- 47 Which words were **most important**? Why?

Help students evaluate their own processes and engage in productive peer interaction

- 35 What do you need to do **next**?
- 36 What have you **accomplished**?
- 37 What are your **strengths and weaknesses**?
- 38 Was your **group participation appropriate and helpful**?



Help students learn to **conjecture, invent, and solve problems**

- 48 What would happen if ___?
- 49 Do you see a **pattern**?
- 50 What are some **possibilities** here?
- 51 Where could you find the **information** you need?
- 52 How would you **check your steps** or your answer?
- 53 What **did not work**?
- 54 How is your solution method the **same as or different from** [student]'s method?
- 55 Other than retracing your steps, **how can you determine** if your answers are appropriate?
- 56 How did you **organize** the information? Do you have a **record**?
- 57 How could you solve this using **tables, lists, pictures, diagrams**, etc.?
- 58 What have you tried? What **steps** did you take?
- 59 How would it look if you used this **model** or these **materials**?
- 60 How would you draw a **diagram or make a sketch** to solve the problem?
- 61 Is there **another possible answer**? If so, explain.
- 62 Is there **another way to solve** the problem?
- 63 Is there **another model** you could use to solve the problem?
- 64 Is there anything you've **overlooked**?
- 65 **How did you think** about the problem?
- 66 What was your **estimate or prediction**?
- 67 How **confident** are you in your answer?
- 68 **What else** would you like to know?
- 69 What do you think comes **next**?
- 70 Is the solution **reasonable**, considering the context?
- 71 Did you have a **system**? Explain it.
- 72 Did you have a **strategy**? Explain it.
- 73 Did you have a **design**? Explain it.



Help students learn to connect mathematics, its ideas, and its application

- 74 What is the **relationship** between ___ and ___?
- 75 Have we ever solved a problem **like this before**?
- 76 What uses of mathematics did you find in the **newspaper** last night?
- 77 What is the **same**?
- 78 What is **different**?
- 79 Did you use skills or build on concepts that were **not necessarily mathematical**?
- 80 Which **skills or concepts** did you use?
- 81 What **ideas** have we explored before that were useful in solving this problem?

- 82 Is there a **pattern**?
- 83 **Where else** would this strategy be useful?
- 84 How does this **relate** to ___?
- 85 Is there a **general rule**?
- 86 Is there a **real-life situation** where this could be used?
- 87 How would your method work with **other problems**?
- 88 What other problem does this seem to **lead to**?

Help students persevere

- 95 What was **one thing you learned** (or two, or more)?
- 96 Did you **notice any patterns**? If so, describe them.
- 97 What **mathematics topics** were used in this investigation?
- 98 What were the **mathematical ideas** in this problem?
- 99 What is mathematically **different about these two situations**?
- 100 What are the **variables** in this problem? What stays **constant**?

- 89 Have you tried making a **guess**?
- 90 **What else** have you tried?
- 91 Would **another method** work as well or better?
- 92 Is there **another way** to draw, explain, or say that?
- 93 Give me another **related problem**. Is there an easier problem?
- 94 How would you **explain** what you know right now?

Help students focus on the mathematics from activities

Conceptual Understanding

Students demonstrate conceptual understanding in mathematics when they provide evidence that they can:

- recognize, label, and generate examples of concepts;
- use and interrelate models, diagrams, manipulatives, and varied representations of concepts;
- identify and apply principles; know and apply facts and definitions;
- compare, contrast, and integrate related concepts and principles; and
- recognize, interpret, and apply the signs, symbols, and terms used to represent concepts.

Conceptual understanding reflects a student's ability to reason in settings involving the careful application of concept definitions, relations, or representations of either.

Procedural Fluency

Procedural fluency is the ability to:

- apply procedures accurately, efficiently, and flexibly;
- to transfer procedures to different problems and contexts;
- to build or modify procedures from other procedures; and
- to recognize when one strategy or procedure is more appropriate to apply than another.

Procedural fluency is more than memorizing facts or procedures, and it is more than understanding and being able to use one procedure for a given situation. Procedural fluency builds on a foundation of conceptual understanding, strategic reasoning, and problem solving (NGA Center & CCSSO, 2010; NCTM, 2000, 2014). Research suggests that once students have memorized and practiced procedures that they do not understand, they have less motivation to understand their meaning or the reasoning behind them (Hiebert, 1999). Therefore, the development of students' conceptual understanding of procedures should precede and coincide with instruction on procedures.

Math Fact Fluency: Automaticity

Students who possess math fact fluency can recall math facts with automaticity. Automaticity is the ability to do things without occupying the mind with the low-level details required, allowing it to become an automatic response pattern or habit. It is usually the result of learning, repetition, and practice.

K-2 Math Fact Fluency Expectation

K.OA.5 Add and Subtract within 5.

1.OA.6 Add and Subtract within 10.

2.OA.2 Add and Subtract within 20.

Math Fact Fluency: Fluent Use of Mathematical Strategies

First and second grade students are expected to solve addition and subtraction facts using a variety of strategies fluently.

1.OA.6 Add and subtract within 20, demonstrating fluency for addition and subtraction within 10.

Use strategies such as:

- counting on; making ten (e.g., $8 + 6 = 8 + 2 + 4 = 10 + 4 = 14$);
- decomposing a number leading to a ten (e.g., $13 - 4 = 13 - 3 - 1 = 10 - 1 = 9$);
- using the relationship between addition and subtraction; and
- creating equivalent but easier or known sums.

2.NBT.7 Add and subtract within 1000, using concrete models or drawings and strategies based on:

- place value,
- properties of operations, and/or
- the relationship between addition and subtraction;

Evidence of Student Thinking

Effective classroom instruction and more importantly, improving student performance, can be accomplished when educators know how to elicit evidence of students' understanding on a daily basis. Informal and formal methods of collecting evidence of student understanding enable educators to make positive instructional changes. An educators' ability to understand the processes that students use helps them to adapt instruction allowing for student exposure to a multitude of instructional approaches, resulting in higher achievement. By highlighting student thinking and misconceptions, and eliciting information from more students, all teachers can collect more representative evidence and can therefore better plan instruction based on the current understanding of the entire class.

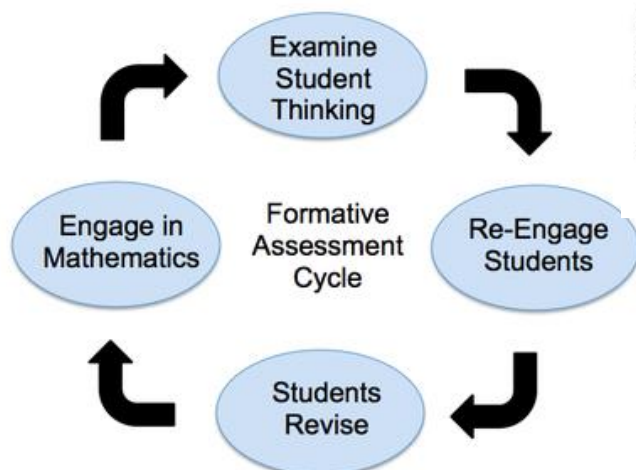
Mathematical Proficiency

To be mathematically proficient, a student must have:

- Conceptual understanding: comprehension of mathematical concepts, operations, and relations;
- Procedural fluency: skill in carrying out procedures flexibly, accurately, efficiently, and appropriately;
- Strategic competence: ability to formulate, represent, and solve mathematical problems;
- Adaptive reasoning: capacity for logical thought, reflection, explanation, and justification;
- Productive disposition: habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

Evidence should:

- Provide a window in student thinking;
- Help teachers to determine the extent to which students are reaching the math learning goals; and
- Be used to make instructional decisions during the lesson and to prepare for subsequent lessons.



Formative assessment is an essentially interactive process, in which the teacher can find out whether what has been taught has been learned, and if not, to do something about it. Day-to-day formative assessment is one of the most powerful ways of improving learning in the mathematics classroom.

(Wiliam 2007, pp. 1054; 1091)

Connections to the Mathematical Practices

Student Friendly Connections to the Mathematical Practices

1. I can solve problems without giving up.
2. I can think about numbers in many ways.
3. I can explain my thinking and try to understand others.
4. I can show my work in many ways.
5. I can use math tools and tell why I choose them.
6. I can work carefully and check my work.
7. I can use what I know to solve new problems.
8. I can discover and use short cuts.

The **Standards for Mathematical Practice** describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

Make sense of problems and persevere in solving them

1 Mathematically proficient students in First Grade continue to develop the ability to focus attention, test hypotheses, take reasonable risks, remain flexible, try alternatives, exhibit self-regulation, and persevere (Copley, 2010). As the teacher uses thoughtful questioning and provides opportunities for students to share thinking, First Grade students become conscious of what they know and how they solve problems. They make sense of task-type problems, find an entry point or a way to begin the task, and are willing to try other approaches when solving the task. They ask themselves, “Does this make sense?” First Grade students’ conceptual understanding builds from their experiences in Kindergarten as they continue to rely on concrete manipulatives and pictorial representations to solve a problem, eventually becoming fluent and flexible with mental math as a result of these experiences..

Reason abstractly and quantitatively

2 Mathematically proficient students in First Grade recognize that a number represents a specific quantity. They use numbers and symbols to represent a problem, explain thinking, and justify a response. For example, when solving the problem: “There are 60 children on the playground. Some children line up. There are 20 children still on the playground. How many children lined up?” first grade students may write $20 + 40 = 60$ to indicate a Think-Addition strategy. Other students may illustrate a counting-on by tens strategy by writing $20 + 10 + 10 + 10 + 10 = 60$. The numbers and equations written illustrate the students’ thinking and the strategies used, rather than how to simply compute, and how the story is decontextualized as it is represented abstractly with symbols.

3	<p>Construct viable arguments and critique the reasoning of others</p> <p>Mathematically proficient students in First Grade continue to develop their ability to clearly express, explain, organize and consolidate their math thinking using both verbal and written representations. Their understanding of grade appropriate vocabulary helps them to construct viable arguments about mathematics. For example, when justifying why a particular shape isn't a square, a first grade student may hold up a picture of a rectangle, pointing to the various parts, and reason, "It can't be a square because, even though it has 4 sides and 4 angles, the sides aren't all the same size." In a classroom where risk-taking and varying perspectives are encouraged, mathematically proficient students are willing and eager to share their ideas with others, consider other ideas proposed by classmates, and question ideas that don't seem to make sense.</p>
4	<p>Model with mathematics</p> <p>Mathematically proficient students in First Grade model real-life mathematical situations with a number sentence or an equation, and check to make sure that their equation accurately matches the problem context. They also use tools, such as tables, to help collect information, analyze results, make conclusions, and review their conclusions to see if the results make sense and revising as needed.</p>
5	<p>Use appropriate tools strategically</p> <p>Mathematically proficient students in First Grade have access to a variety of concrete (e.g. 3-dimensional solids, ten frames, number balances, number lines) and technological tools (e.g., virtual manipulatives, calculators, interactive websites) and use them to investigate mathematical concepts. They select tools that help them solve and/or illustrate solutions to a problem. They recognize that multiple tools can be used for the same problem- depending on the strategy used. For example, a child who is in the counting stage may choose connecting cubes to solve a problem. While, a student who understands parts of number, may solve the same problem using ten-frames to decompose numbers rather than using individual connecting cubes. As the teacher provides numerous opportunities for students to use educational materials, first grade students' conceptual understanding and higher order thinking skills are developed</p>

6	<p>Attend to precision</p> <p>Mathematically proficient students in First Grade attend to precision in their communication, calculations, and measurements. They are able to describe their actions and strategies clearly, using grade-level appropriate vocabulary accurately. Their explanations and reasoning regarding their process of finding a solution becomes more precise. In varying types of mathematical tasks, first grade students pay attention to details as they work. For example, as students' ability to attend to position and direction develops, they begin to notice reversals of numerals and self-correct when appropriate. When measuring an object, students check to make sure that there are not any gaps or overlaps as they carefully place each unit end to end to measure the object (iterating length units). Mathematically proficient first grade students understand the symbols they use ($=$, $>$, $<$, a proficient student who is able to attend to precision states, "Four is more than 3" rather than "The alligator eats the four. It's bigger."</p>
7	<p>Look for and make use of structure</p> <p>Mathematically proficient students in First Grade carefully look for patterns and structures in the number system and other areas of mathematics. For example, while solving addition problems using a number balance, students recognize that regardless whether you put the 7 on a peg first and then the 4, or the 4 on first and then the 7, they both equal 11 (commutative property). When decomposing two-digit numbers, students realize that the number of tens they have constructed 'happens' to coincide with the digit in the tens place. When exploring geometric properties, first graders recognize that certain attributes are critical (number of sides, angles), while other properties are not (size, color, orientation)</p>
8	<p>Look for and express regularity in repeated reasoning</p> <p>Mathematically proficient students in First Grade begin to look for regularity in problem structures when solving mathematical tasks. For example, when adding three one-digit numbers and by making tens or using doubles, students engage in future tasks looking for opportunities to employ those same strategies. Thus, when solving $8+7+2$, a student may say, "I know that 8 and 2 equal 10 and then I add 7 more. That makes 17. It helps to see if I can make a 10 out of 2 numbers when I start." Further, students use repeated reasoning while solving a task with multiple correct answers. For example, in the</p>

task “There are 12 crayons in the box. Some are red and some are blue. How many of each could there be?” First Grade students realize that the 12 crayons could include 6 of each color ($6+6 = 12$), 7 of one color and 5 of another ($7+5 = 12$), etc. In essence, students repeatedly find numbers that add up to 12.

Effective Mathematics Teaching Practices

Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.

Implement tasks that promote reasoning and problem solving. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.

Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.

Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.

Pose purposeful questions. Effective teaching of mathematics uses purposeful questions to assess and advance students’ reasoning and sense making about important mathematical ideas and relationships.

Build procedural fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.

Support productive struggle in learning mathematics. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.

Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

5 Practices for Orchestrating Productive Mathematics Discussions

Practice	Description/ Questions
1. Anticipating	<p>What strategies are students likely to use to approach or solve a challenging high-level mathematical task?</p> <p>How do you respond to the work that students are likely to produce?</p> <p>Which strategies from student work will be most useful in addressing the mathematical goals?</p>
2. Monitoring	<p>Paying attention to what and how students are thinking during the lesson.</p> <p>Students working in pairs or groups</p> <p>Listening to and making note of what students are discussing and the strategies they are using</p> <p>Asking students questions that will help them stay on track or help them think more deeply about the task. (Promote productive struggle)</p>
3. Selecting	<p>This is the process of deciding the <i>what</i> and the <i>who</i> to focus on during the discussion.</p>
4. Sequencing	<p>What order will the solutions be shared with the class?</p>
5. Connecting	<p>Asking the questions that will make the mathematics explicit and understandable.</p> <p>Focus must be on mathematical meaning and relationships; making links between mathematical ideas and representations.</p>

MATH CENTERS/ WORKSTATIONS

Math workstations allow students to engage in authentic and meaningful hands-on learning. They often last for several weeks, giving students time to reinforce or extend their prior instruction. Before students have an opportunity to use the materials in a station, introduce them to the whole class, several times. Once they have an understanding of the concept, the materials are then added to the work stations.

Station Organization and Management Sample

Teacher A has 12 containers labeled 1 to 12. The numbers correspond to the numbers on the rotation chart. She pairs students who can work well together, who have similar skills, and who need more practice on the same concepts or skills. Each day during math work stations, students use the center chart to see which box they will be using and who their partner will be. Everything they need for their station will be in their box. **Each station is differentiated.** If students need more practice and experience working on numbers 0 to 10, those will be the only numbers in their box. If they are ready to move on into the teens, then she will place higher number activities into the box for them to work with.



In the beginning there is a lot of prepping involved in gathering, creating, and organizing the work stations. However, once all of the initial work is complete, the stations are easy to manage. Many of her stations stay in rotation for three or four weeks to give students ample opportunity to master the skills and concepts.

Read *Math Work Stations* by Debbie Diller.

In her book, she leads you step-by-step through the process of implementing work stations.

MATH WORKSTATION INFORMATION CARD

Math Workstation: _____

Time:

NJSLS:

Objective(s): By the end of this task, I will be able to:

- _____
- _____
- _____

Task(s):

- _____
- _____
- _____
- _____

Exit Ticket:

- _____
- _____
- _____

MATH WORKSTATION SCHEDULE

Week of: _____

DAY	Technology Lab	Problem Solving Lab	Fluency Lab	Math Journal	Small Group Instruction
Mon.	Group ____	Group ____	Group ____	Group ____	BASED ON CURRENT OBSERVATIONAL DATA
Tues.	Group ____	Group ____	Group ____	Group ____	
Wed.	Group ____	Group ____	Group ____	Group ____	
Thurs.	Group ____	Group ____	Group ____	Group ____	
Fri.	Group ____	Group ____	Group ____	Group ____	

INSTRUCTIONAL GROUPING

	GROUP A		GROUP B
1		1	
2		2	
3		3	
4		4	
5		5	
6		6	
	GROUP C		GROUP D
1		1	
2		2	
3		3	
4		4	
5		5	
6		6	

First Grade PLD Rubric

Got It		Not There Yet		
Evidence shows that the student essentially has the target concept or big math idea.		Student shows evidence of a major misunderstanding, incorrect concepts or procedure, or a failure to engage in the task.		
PLD Level 5: 100% Distinguished command	PLD Level 4: 89% Strong Command	PLD Level 3: 79% Moderate Command	PLD Level 2: 69% Partial Command	PLD Level 1: 59% Little Command
<p>Student work shows distinct levels of understanding of the mathematics.</p> <p>Student constructs and communicates a complete response based on explanations/reasoning using the:</p> <ul style="list-style-type: none"> • Tools: <ul style="list-style-type: none"> ○ Manipulatives ○ Five Frame ○ Ten Frame ○ Number Line ○ Part-Part-Whole Model • Strategies: <ul style="list-style-type: none"> ○ Drawings ○ Counting All ○ Count On/Back ○ Skip Counting ○ Making Ten ○ Decomposing Number • Precise use of math vocabulary <p>Response includes an efficient and logical progression of mathematical reasoning and understanding.</p>	<p>Student work shows strong levels of understanding of the mathematics.</p> <p>Student constructs and communicates a complete response based on explanations/reasoning using the:</p> <ul style="list-style-type: none"> • Tools: <ul style="list-style-type: none"> ○ Manipulatives ○ Five Frame ○ Ten Frame ○ Number Line ○ Part-Part-Whole Model • Strategies: <ul style="list-style-type: none"> ○ Drawings ○ Counting All ○ Count On/Back ○ Skip Counting ○ Making Ten ○ Decomposing Number • Precise use of math vocabulary <p>Response includes a logical progression of mathematical reasoning and understanding.</p>	<p>Student work shows moderate levels of understanding of the mathematics.</p> <p>Student constructs and communicates a complete response based on explanations/reasoning using the:</p> <ul style="list-style-type: none"> • Tools: <ul style="list-style-type: none"> ○ Manipulatives ○ Five Frame ○ Ten Frame ○ Number Line ○ Part-Part-Whole Model • Strategies: <ul style="list-style-type: none"> ○ Drawings ○ Counting All ○ Count On/Back ○ Skip Counting ○ Making Ten ○ Decomposing Number • Precise use of math vocabulary <p>Response includes a logical but incomplete progression of mathematical reasoning and understanding. Contains minor errors.</p>	<p>Student work shows partial understanding of the mathematics.</p> <p>Student constructs and communicates an incomplete response based on student's attempts of explanations/ reasoning using the:</p> <ul style="list-style-type: none"> • Tools: <ul style="list-style-type: none"> ○ Manipulatives ○ Five Frame ○ Ten Frame ○ Number Line ○ Part-Part-Whole Model • Strategies: <ul style="list-style-type: none"> ○ Drawings ○ Counting All ○ Count On/Back ○ Skip Counting ○ Making Ten ○ Decomposing Number • Precise use of math vocabulary <p>Response includes an incomplete or illogical progression of mathematical reasoning and understanding.</p>	<p>Student work shows little understanding of the mathematics.</p> <p>Student attempts to construct and communicates a response using the:</p> <ul style="list-style-type: none"> • Tools: <ul style="list-style-type: none"> ○ Manipulatives ○ Five Frame ○ Ten Frame ○ Number Line ○ Part-Part-Whole Model • Strategies: <ul style="list-style-type: none"> ○ Drawings ○ Counting All ○ Count On/Back ○ Skip Counting ○ Making Ten ○ Decomposing Number • Precise use of math vocabulary <p>Response includes limited evidence of the progression of mathematical reasoning and understanding.</p>
5 points	4 points	3 points	2 points	1 point

DATA DRIVEN INSTRUCTION

Formative assessments inform instructional decisions. Taking inventories and assessments, observing reading and writing behaviors, studying work samples and listening to student talk are essential components of gathering data. When we take notes, ask questions in a student conference, lean in while a student is working or utilize a more formal assessment we are gathering data. Learning how to take the data and record it in a meaningful way is the beginning of the cycle.

Analysis of the data is an important step in the process. What is this data telling us? We must look for patterns, as well as compare the notes we have taken with work samples and other assessments. We need to decide what are the strengths and needs of individuals, small groups of students and the entire class. Sometimes it helps to work with others at your grade level to analyze the data.

Once we have analyzed our data and created our findings, it is time to make informed instructional decisions. These decisions are guided by the following questions:

- What mathematical practice(s) and strategies will I utilize to teach to these needs?
- What sort of grouping will allow for the best opportunity for the students to learn what it is I see as a need?
- Will I teach these strategies to the whole class, in a small guided group or in an individual conference?
- Which method and grouping will be the most effective and efficient? What specific objective(s) will I be teaching?

Answering these questions will help inform instructional decisions and will influence lesson planning.

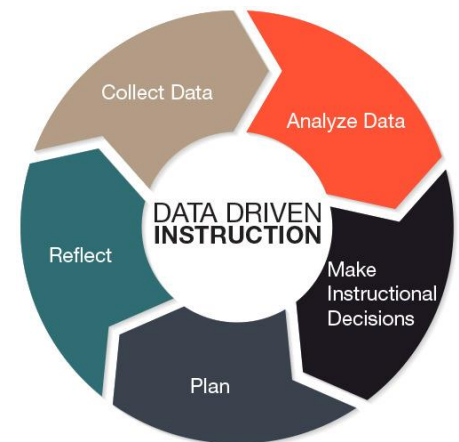
Then we create our instructional plan for the unit/month/week/day and specific lessons.

It's important now to reflect on what you have taught.

Did you observe evidence of student learning through your checks for understanding, and through direct application in student work?

What did you hear and see students doing in their reading and writing?

Now it is time to begin the analysis again.



Data Analysis Form

School: _____

Teacher: _____

Date: _____

Assessment: _____

NJSLS: _____

GROUPS (STUDENT INITIALS)	SUPPORT PLAN	PROGRESS
MASTERED (86% - 100%) (PLD 4/5):		
DEVELOPING (67% - 85%) (PLD 3):		
INSECURE (51%-65%) (PLD 2):		
BEGINNING (0%-50%) (PLD 1):		

MATH PORTFOLIO EXPECTATIONS

The **Student Assessment Portfolios for Mathematics** are used as a means of documenting and evaluating students' academic growth and development over time and in relation to the CCSS-M. The September task entry(-ies) should reflect the prior year content and *can serve* as an additional baseline measure.

All tasks contained within the **Student Assessment Portfolios** should be aligned to NJSL and be “practice forward” (closely aligned to the Standards for Mathematical Practice).

Four (4) or more additional tasks will be included in the **Student Assessment Portfolios** for Student Reflection and will be labeled as such.

K-2 GENERAL PORTFOLIO EXPECTATIONS:

- Tasks contained within the Student Assessment Portfolios are “practice forward” and denoted as “Individual”, “Partner/Group”, and “Individual w/Opportunity for Student Interviews¹.”
- Each Student Assessment Portfolio should contain a “Task Log” that documents all tasks, standards, and rubric scores aligned to the performance level descriptors (PLDs).
- Student work should be attached to a completed rubric; with appropriate teacher feedback on student work.
- Students will have multiple opportunities to revisit certain standards. Teachers will capture each additional opportunity “as a new and separate score” in the task log.
- A 2-pocket folder for each Student Assessment Portfolio is *recommended*.
- All Student Assessment Portfolio entries should be scored and recorded as an Authentic Assessment grade (25%)².
- All Student Assessment Portfolios must be clearly labeled, maintained for all students, inclusive of constructive teacher and student feedback and accessible for review.

GRADES K-2

Student Portfolio Review

Provide students the opportunity to review and evaluate their portfolio at various points throughout the year; celebrating their progress and possibly setting goals for future growth. During this process, students should retain ALL of their current artifacts in their Mathematics Portfolio

1st Grade Authentic Performance Task: Pet Snake

Pet Snake Performance Task

Name: _____ Teacher: _____ Date: _____

- a. The class had a pet snake. It was 14 inches long. It grew 3 more inches. How long is it now?

- b. The class had a pet snake. It was 14 inches long. It grew a few more inches. Now it is 17 inches long. How many inches did it grow?

- c. The class had a pet snake. It grew 3 more inches. Now it is 17 inches long. How long was it to start?

Pet Snake

Standard(s)	1.OA.1 Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.
Materials	copy of the problem, pencil

Level 5: Distinguished Command	Level 4: Strong Command	Level 3: Moderate Command	Level 2: Partial Command	Level 1: No Command
<p>Student correctly answers and clearly constructs and communicates a complete response based on explanations/reasoning using:</p> <ul style="list-style-type: none"> • Strategies based on place value, counting on, making a ten, mental math strategies, etc. • Properties of addition <p>Response includes an efficient and logical progression of steps.</p> <p>Strategy and execution meet the content, process, and qualitative demands of the task or concept. Student can communicate ideas.</p>	<p>Student correctly answers and clearly constructs and communicates a complete response with one minor error based on explanations/reasoning using:</p> <ul style="list-style-type: none"> • Strategies based on place value, counting on, making a ten, mental math strategies, etc. • Properties of addition <p>Response includes a logical progression of steps</p> <p>May have minor errors that do not impact the mathematics.</p>	<p>Student answers, clearly constructs, and communicates a complete response with minor errors based on explanations/reasoning using:</p> <ul style="list-style-type: none"> • Strategies based on place value, counting on, making a ten, mental math strategies, etc. • Properties of addition <p>Response includes a logical but incomplete progression of steps. Minor calculation errors.</p>	<p>The task is attempted, and some mathematical effort is made. There may be fragments of accomplishment but little success.</p> <ul style="list-style-type: none"> • Strategies based on place value, counting on, making a ten, mental math strategies, etc. • Properties of addition <p>Response includes an incomplete or illogical progression of steps.</p>	<p>The student shows no work or justification. Further teaching is required.</p>

1st Grade Authentic Performance Task: 20 Tickets

20 Tickets Performance Task

Name: _____ Teacher: _____ Date: _____

Bo bought 20 tickets to play games at Family Fun Night at his school. He wants to play each game at least once. He needs to use all of his tickets. How many times might he play each game?

Game	Number of Tickets Needed
Ring Toss	1
Putt-Putt Golf	2
Soccer Kick	3
Moonwalk	5

20 Tickets

Standard(s)	1.OA.1 Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.
Materials	20 counters or cubes, copy of the problem, pencil

The purpose of the task is for students to add and subtract within 20 (1.OA.1) and represent complex addition problems with an equation to increase their understanding of and flexibility with the equals sign (1.OA.7).

There are multiple solutions, and each pair of students should find more than one. The students can use the counters or linking cubes to represent each ticket needed to do each game, but then they should be encouraged to draw a picture to represent their work so there is a record of their thought process. Students who are comfortable with symbolic representations can record their solutions using equations.

The problem can be differentiated by using either a smaller or larger number of tickets. An extension would also be to have the students find the greatest number of times the games could be played to still do all games at least once. Another would be to ask if they can play each game twice and justify their thinking and solution.

Level 5: Distinguished Command	Level 4: Strong Command	Level 3: Moderate Command	Level 2: Partial Command	Level 1: No Command
<p>Student correctly answers and clearly constructs and communicates a complete response based on explanations/reasoning using:</p> <ul style="list-style-type: none"> Strategies based on place value, counting on, making a ten, mental math strategies, etc. <p>Response includes an efficient and logical progression of steps.</p> <p>Strategy and execution meet the content, process, and qualitative demands of the task or concept. Student can communicate ideas.</p>	<p>Student correctly answers and clearly constructs and communicates a complete response with one minor error based on explanations/reasoning using:</p> <ul style="list-style-type: none"> Strategies based on place value, counting on, making a ten, mental math strategies, etc. <p>Response includes a logical progression of steps</p> <p>May have minor errors that do not impact the mathematics.</p>	<p>Student answers, clearly constructs, and communicates a complete response with minor errors based on explanations/reasoning using:</p> <ul style="list-style-type: none"> Strategies based on place value, counting on, making a ten, mental math strategies, etc. <p>Response includes a logical but incomplete progression of steps. Minor calculation errors.</p>	<p>The task is attempted, and some mathematical effort is made. There may be fragments of accomplishment but little success.</p> <ul style="list-style-type: none"> Strategies based on place value, counting on, making a ten, mental math strategies <p>Response includes an incomplete or illogical progression of steps.</p>	<p>The student shows no work or justification. Further teaching is required.</p>

Resources

Number Book Assessment Link: <http://investigations.terc.edu/>

Model Curriculum- <http://www.nj.gov/education/modelcurriculum/>

Georgia Department of Education: Games to be played at centers with a partner or small group. <http://ccgpsmathematicsk-5.wikispaces.com/Kindergarten>

Engage NY: *For additional resources to be used during centers or homework.

<https://www.engageny.org/sites/default/files/resource/attachments/math-gk-m1-full-module.pdf>

Add/ Subtract Situation Types: Darker Shading indicates Kindergarten expectations
<https://achievethecore.org/content/upload/Add%20Subtract%20Situation%20Types.pdf>

Math in Focus PD Videos: <https://www->

[k6.thinkcentral.com/content/hsp/math/hspmath/common/mif_pd_vid/9780547760346_te/index.html](https://www-)

Suggested Literature

Fish Eyes by, Lois Ehlert

Ten Little Puppies by, Elena Vazquez

Zin! Zin! Zin! A Violin! by, Lloyd Moss

My Granny Went to the Market by, Stella Blackstone and Christopher Corr

Anno's Counting Book by, Mitsumasa Anno

Chicka, Chicka, 1,2,3 by, Bill Martin Jr.; Michael Sampson; Lois Ehlert

How Dinosaurs Count to 10 by Jane Yolen and Mark Teague

10 Little Rubber Ducks by Eric Carle

Ten Black Dots by Donald Crews

Mouse Count by Ellen Stoll Walsh

Count! by Denise Fleming

21st Century Career Ready Practices

- CRP1. Act as a responsible and contributing citizen and employee.
- CRP2. Apply appropriate academic and technical skills.
- CRP3. Attend to personal health and financial well-being.
- CRP4. Communicate clearly and effectively and with reason.
- CRP5. Consider the environmental, social and economic impacts of decisions.
- CRP6. Demonstrate creativity and innovation.
- CRP7. Employ valid and reliable research strategies.
- CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.
- CRP9. Model integrity, ethical leadership and effective management.
- CRP10. Plan education and career paths aligned to personal goals.
- CRP11. Use technology to enhance productivity.
- CRP12. Work productively in teams while using cultural global competence.

For additional details see **21st Century Career Ready Practices** .

References

“Eureka Math” *Great Minds*. 2018 < <https://greatminds.org/account/products>>